# Session 2

## Data-Driven Decision Making with and without Probabilities

1. Mr. Murong is considering enrolling his son into four different enrichment classes. Some classes start off cheap, but gets more expensive as time continues (e.g. more expensive equipment needed). On the other hand, due to the availability of grants or competition awards, some activities actually get cheaper over time. He is faced with four decisions alternatives and three states of nature, as shown in the following cost table:

**Total Cost of participating in enrichment classes**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **State of Nature** | | |
| **Decision**  **Alternatives** | *s*1: Stop at  7 years old | *s*2: Stop at  16 years old | *s*3: Continue  after 16 years old |
| G: Gymnastics | $4000 | $8600 | $7200 |
| A: Air Rifle | $6000 | $8000 | $6500 |
| S: Sailing | $5000 | $9200 | $15000 |
| C: Chess | $3000 | $11000 | $5000 |

E.g. If Mr. Murong’s son only stops participating in gymnastics at 16 years old, Mr. Murong would have spent $8600 on his classes and involvement in gymnastics.

1. What is the recommended decision using the optimistic, conservative and minimax regret approaches?

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| --- | --- | --- |
| **Decision Alternatives** | **Optimistic** | **Conversative** |
| G: Gymnastics | 4000 | 8600 |
| A: Air Rifle | 6000 | 8000 |
| S: Sailing | 5000 | 15000 |
| C: Chess | 3000 | 11000 |

Optimistic approach should choose: **Chess**

Conservative approach choose: **Air Rifle**

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| --- | --- | --- | --- | --- |
| **Decision**  **Alternatives** | ***s*1: Stop at**  **7 years old** | ***s*2: Stop at**  **16 years old** | ***s*3: Continue**  **after 16 years old** | **Maximum Regret** |
| G: Gymnastics | 1000 | 600 | 2200 | 2200 |
| A: Air Rifle | 3000 | 0 | 1500 | 3000 |
| S: Sailing | 2000 | 1200 | 10000 | 10000 |
| C: Chess | 0 | 3000 | 0 | 3000 |

MINIMAX regret approach should choose: **Gymnastics**

1. Mr. Murong research on the number of people in Singapore participating in each activity, and calculated the following probabilities:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Probabilities** | | |
| **Decision**  **Alternatives** | *s*1: Stop at  7 years old | *s*2: Stop at  16 years old | *s*3: Continue  after 16 years old |
| G: Gymnastics | P(*s*1 | G) = 0.10 | P(*s*2 | G) = 0.80 | P(*s*3 | G) = 0.10 |
| A: Air Rifle | P(*s*1 | A) = 0.20 | P(*s*2 | A) = 0.65 | P(*s*3 | A) = 0.15 |
| S: Sailing | P(*s*1 | S) = 0.70 | P(*s*2 | S) = 0.25 | P(*s*3 | S) = 0.05 |
| C: Chess | P(*s*1 | C) = 0.30 | P(*s*2 | C) = 0.40 | P(*s*3 | C) = 0.30 |

With these probabilities, calculate the best decision using the expected value approach.

EV(G) = 0.10(4000) + 0.80(8600) + 0.10(7200) = 8000

EV(A) = 0.20(6000) + 0.65(8000) + 0.15(6500) = 7375

EV(S) = 0.70(5000) + 0.25(9200) + 0.05(15000) = 6550

EV(V) = 0.30(3000) + 0.40(11000) + 0.30(5000) = 6800

Hence, he should choose **Sailing**.

The expected value for taking the Sailing class is $6550 (lowest).

That means whatever his son will stop attending the class at 7, 16 or 16 after 16 years old, the estimated financial commitment is the lowest among the others.

1. Suppose that you are given a decision situation with three possible states of nature: *s*1, *s*2, and *s*3. The prior probabilities are P(*s*1) = 0.2, P(*s*2) = 0.5, and P(*s*3) = 0.3. With sample information *I*, P(*I* | *s*1) = 0.1, P(*I* | *s*2) = 0.05, and P(*I* | *s*3) = 0.2. Compute the revised or posterior probabilities: P(*s*1 | *I*), P(*s*2 | *I*), and P(*s*3 | *I*).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| States of Nature *s*1 | Prior Probabilities P(*s*1) | Conditional Probabilities  P(*I* | *s*1) | Joint Probabilities P(*s*1 n *I*) | Revised/Posterior Probabilities P(*s*1 | *I*) |
| *S1* | 0.2 | 0.1 | (0.2)(0.1) = 0.02 | 0.02/0.105 = 0.1905 |
| *S2* | 0.5 | 0.05 | (0.5)(0.05) =0.025 | 0.025/0.105 = 0.2381 |
| *S3* | 0.3 | 0.2 | (0.3)(0.2) = 0.06 | 0.06/0.105 = 0.5714 |
|  | Sum = 1.0  (for checking) |  | P(*I*) = 0.105 | Sum = 1.00 |

1. To save on expenses, Rosa and Jerry agreed to form a carpool for traveling to and from work. Rosa preferred to use the somewhat longer but more consistent Buangkok Avenue. Although Jerry preferred the quicker expressway, he agreed with Rosa that they should take Buangkok Avenue if the expressway had a traffic jam. The following cost table provides the one-way time estimate in minutes for travelling to and from work:

|  |  |  |
| --- | --- | --- |
|  | State of Nature | |
| **Decision**  **Alternatives** | *s*1: Expressway Open | *s*2: Expressway Jammed |
| Buangkok Avenue, *d*1 | 30 | 30 |
| Expressway, *d*2 | 25 | 45 |

Based on their experience with traffic problems, Rosa and Jerry agreed on a 0.15 probability that the expressway would be jammed. P(*s*1) = 0.15, P(*s*2) = 1 – 0.15 = 0.85

In addition, they agreed that weather seemed to affect the traffic conditions on the expressway. Let

|  |
| --- |
| *C* = clear |
| *O* = overcast |
| *R* = rain |

The following conditional probabilities apply:

|  |  |  |
| --- | --- | --- |
| P(*C* | *s*1) = 0.8 | P(*O* | *s*1) = 0.2 | P(*R* | *s*1) = 0.0 |
| P(*C* | *s*2) = 0.1 | P(*O* | *s*2) = 0.3 | P(*R* | *s*2) = 0.6 |

1. Use Bayes’ theorem for probability revision to compute the probability of each weather condition and the conditional probability of the expressway open, s1, or jammed, s2, given each weather condition.
2. Show the decision tree for this problem.
3. What is the optimal decision strategy, and what is the expected travel time?

(a) Overcast

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| States of Nature *s*1 | Prior Probabilities P(*s*1) | Conditional Probabilities  P(C| *s*1) | Joint Probabilities  P(C n *s*1) | Revised/Posterior Probabilities P(*s*1 | C) |
| *S1 (open)* | 0.85 | 0.2 | (0.85)(0.2) = 0.17 | 0.17/0.215= 0.791 |
| *S2 (jammed)* | 0.15 | 0.3 | (0.15)(0.3) = 0.045 | 0.045/0.215= 0.209 |
|  | Sum = 1.00  (for checking) |  | P(C) = 0.17 + 0.045 = 0.215 | Sum = 1.00 |

Rain

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *S1 (open)* | 0.85 | 0.0 | 0.0 | 0.0 |
| *S2 (jammed)* | 0.15 | 0.6 | 0.09 | 1.0 |
|  | Sum = 1.00  (for checking) |  | P(C) = 0.09 | Sum = 1.00 |

Clear

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *S1 (open)* | 0.85 | 0.8 | (0.85)(0.8) = 0.68 | 0.68/0.695 = 0.9784172662 |
| *S2 (jammed)* | 0.15 | 0.1 | (0.15)(0.1) = 0.015 | 0.015/0.695 = 0.0215827338 |
|  | Sum = 1.00  (for checking) |  | P(C) = 0.68 + 0.015 = 0.695 | Sum = 1.00 |

(b)